A Simple Formula for Calculation of the Total Dose in Fractionated Radiotherapy

Fowler (4), Liversage (6), and Ellis (2) have recently published excellent reviews of clinical models and formulae. The conclusion drawn from their discussions was that, for lack of accurate experimental methods and results, the formulae coupling together dose, time, and number of fractions need to be considered to some extent as rules of thumb.

With this conclusion as a starting point, more liberty could be taken in approximations for derivation of the formulae proposed. A firm foundation exists for use of the equation \( D = D_1 A^k \) in analysis of the change in the total treatment dose \( D \) (rads) as a function of the number of fractions \( N \), the overall treatment time \( T \) (days), or clinical results involving both of them. \( A \) can be replaced by \( N \) or \( T \), or by a symbol representing a mixture of them. The slope \( k \) of the straight line in the log-log coordinate system is analyzable from experimental results. \( D_1 \) is a constant (nominal standard dose, ret dose) representing the real or assumed single dose, which in principle induces the same radiobiological effects as the method of treatment applied.

Without going into details, we can conclude that slope value \( k \) varies from case to case, from values of 0.03 up to 0.40 (7), with most values falling within the range 0.24 to 0.34 (2). As an average value, 0.27 is acceptable, if a rule of thumb is used. Moreover, the constant \( D_1 \) can be assumed to remain approximately constant on changing from one method of treatment to another (2).

Consequently, the following derivation can be made:

\[
D_{\text{new}} = D_1 \times N_{\text{new}}^{0.27},
\]

which is valid for the “new” method, where \( D_{\text{new}} \) is the total treatment dose (rads) and \( N_{\text{new}} \) is the number of fractions. The same formula is valid for the “old” method applied earlier:

\[
D_{\text{old}} = D_1 \times N_{\text{old}}^{0.27},
\]

where \( D_{\text{old}} \) and \( N_{\text{old}} \) represent the total treatment dose and the number of fractions, respectively, for this method of treatment.

If it is assumed that \( D_1 \) is constant and formula (A) is divided by formula (B), there is obtained:

\[
D_{\text{new}} = \left( \frac{N_{\text{new}}}{N_{\text{old}}} \right)^{0.27} \times D_{\text{old}},
\]

or

\[
D_{\text{new}} = k \times D_{\text{old}},
\]

where \( k = (N_{\text{new}}/N_{\text{old}})^{0.27} \). Table 1 lists the values of \( k \) for \( N \) values from 8 to 30.

Most of the clinical results are based upon “daily” treatments, which means that the comparable total treatment doses also involve a corresponding overall treatment time. Consequently, the slope value 0.27 already in part involves the influence of the overall treatment time. Formula (D) is applicable when the overall treatment is kept constant or the change is small.

When the overall treatment time is considerable, its effect upon the total treatment dose \( D \) is easily calculable by means of the formula proposed by Cohen (1) and Liversage (5):

\[
\Delta D = C(T_{\text{new}} - T_{\text{old}}),
\]

where \( \Delta D \) is the amount of dose that should be added to or subtracted from the total dose \( D \) induced by a change in the overall treatment time \( (T_{\text{new}} - T_{\text{old}}) \). \( T_{\text{new}} \) and \( T_{\text{old}} \) represent the overall treatment times (days) for the new and old methods, respectively. It should be observed that the formula gives the correct sign (+ or —) direct. It can be assumed that the value of the constant \( C \) is 30 rads/day, according to Liversage (6): the change in the overall treatment seldom exceeds 20 days.

The following formula can thus be used:

\[
D_{\text{new}} = k \times D_{\text{old}} + (T_{\text{new}} - T_{\text{old}}) 30 \text{ rads/day.}
\]

The formulae given do not involve any nominal standard dose, ret dose, or similar definitions. The lack of tables of nominal standard doses, and so on, at most radiotherapy centers makes the formulae proposed earlier impractical in use. By reason of differences including those in dosimetry and treatment planning, every center should have its own table. In any case, the center has an old method, which one may wish to change for another new method. Formulae D and F can be useful for this purpose.

For example, the old method consists of 25 fractions given in 30 days, the total dose being 5000 rads. A change to 15 fractions given in 20 days is required. What is the total treatment dose \( D_{\text{new}} \) applicable in the new method?

\[
D_{\text{old}} = 5000 \text{ rads, } D_{\text{new}} = ?, N_{\text{old}} = 25, N_{\text{new}} = 15,
\]

\( T_{\text{old}} = 30 \text{ days, and } T_{\text{new}} = 20 \text{ days} \)

Table 1 gives \( k = 0.87 \).
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\( \text{N}_{\text{new}}^b \)

\[ \text{Number of fractions of the old method.} \]

\[ \text{Number of fractions of the new method.} \]

Table 1

Values of factor \( k \) \( \{ \text{Formulae D and F} \} \) for different numbers of the new and old methods
Observe the assumption that the method is valid only for \( N \) values between 8 and 30.

The accuracy of the formulae has been checked by calculation of the comparable doses given by Fowler (3) and Cohen (1) for numbers of fractions \( N \) between 8 and 30. The results obtained from the doses fall within the range ±3%. This is acceptable in view of the accuracy achievable in the determination of tumor sizes, doses, treatment plans, and so on. It is also good to keep in mind the uncertainty still involved in the experimental and clinical results upon which are based all "models" up to the present day.

Erik Spring

Radiotherapy Clinic
University Central Hospital
Haartmaninkatu 4
0290 Helsinki, Finland

REFERENCES
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Erik Spring


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